Current-phase relations as determinants of superconducting thin-film weak-link \( I-V \) characteristics

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The dc \( I-V \) characteristics of superconducting thin-film Sn-Au proximity bridges and uniform-thickness Sn microbridges have been carefully analyzed as a function of the directly measured current-phase relation (CPR). At sufficiently low dc current and voltage levels where heating and relaxation time effects are not important, the \( I-V \) characteristics are very well described by a shunted weak-link model that includes the proper dc CPR and a shunt resistance.

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A long-standing problem in the study of superconducting weak links is whether or not the current-phase relations (CPR's) which hold for \( i < i_e \) are applicable when \( i > i_e \). That is, can a simple weak-link model which includes the proper dc CPR and a shunt resistance account for measured weak-link \(-V's\) provided voltages and currents are low enough that heating and relaxation time effects are not important?

We have measured \( I-V \) characteristics and CPR's in both Sn uniform-thickness (UT) microbridges and Sn-Au proximity bridges. The details of the CPR and \( I-V \) measurements can be found elsewhere.\(^2\)\(^-\)\(^5\) In the UT bridges the measured multivalued CPR's account for the observed "excess current" \( i_{ex} \), i.e., the time-average supercurrent for \( i > i_e \). In contrast to the theories of Rieger et al.,\(^1\) and Skocpol et al.,\(^6\) we find \( i_{ex} \) both \(<i_{e}\) and \( > i_{e}\) depending upon the particular weak-link CPR. In the proximity bridges near \( T_c \) where a sinusoidal CPR is found we obtain excellent agreement with the complete shunted junction theory of Ambegaokar and Halperin (AH).\(^7\)

UT Sn bridges had multivalued CPR's due in large part to phase winding in the electrodes which lead to the bridge.\(^3\)\(^4\) These CPR's were piecewise linear, with the supercurrent increasing with increasing phase difference \( \Delta \theta \) until a phase slip transition was made to a lower current state where \( \Delta \theta \approx \theta_{c}' \), the average critical phase angle. The equation for the \( I-V \) of a shunted junction model with a piecewise linear CPR is given by

\[
V = -\frac{2\pi R_i}{\theta_{c}'} \ln \left( \frac{i / i_e - 1}{i / i_e + 2\pi / \theta_{c}' + 1} \right),
\]

where \( R \) is the normal shunt resistance. In the case where \( \theta_{c}' = \pi \), Eq. (1) reduces to the model of Deaver and Pierce.\(^8\)

Figure 1 shows typical \( I-V's \) of an Sn UT bridge 1.5 \( \mu \) long, 2.7 \( \mu \) wide, and 1650 \( \AA \) thick, measured at two different temperatures below \( T_c \). The fit of Eq. (1) to the data was obtained by varying \( R, i_e \), and \( \theta_{c}' \). The best-fit CPR's are shown in the insets. These CPR's are very similar to those actually measured in UT bridges made with the same photolithographic mask. The deviation between Eq. (1) and the data near \( i_e \) is due to thermal fluctuation effects similar to those described by

FIG. 1. Measured \( I-V's \) of a UT Sn bridge for two values of \( i_e \). The dotted line shows the fit of a resistively shunted junction model which uses the CPR shown in the inset.
AH. (The AH fluctuation theory is only strictly applicable to weak links with sinusoidal CPR's, so an exact fit in the low-voltage fluctuation regime was not possible.)

We found that the values of $\theta'_c$ which give the best fits to the I-V data are somewhat larger than the values of $\theta'_c$ determined in the current-phase measurements for comparable values of $i_c$. This apparent discrepancy is due to the much greater values of $d(\Delta \theta)/dt$ in the I-V measurements than in the current-phase measurements. For weak links with multivalued CPR's high-current states of the CPR (see insets in Fig. 1) are separated from low-current states by potential energy barriers. These barriers vanish as $\Delta \theta$ approaches a critical value $\theta_c$. In the absence of thermal fluctuations the weak-link supercurrent $i_c$ reaches its maximum value $i_c$ at $\theta_c$. However, at any nonzero temperature, as $\Delta \theta$ is increased in time, thermal fluctuations will drive the system over the barrier to a lower supercurrent state while $\Delta \theta$ is still less than $\theta_c$. These fluctuation-driven transitions occur over a distribution in $\Delta \theta$ centered about $\theta'_c$. The value of $\theta'_c$ depends upon the rate at which $\Delta \theta$ is increased, with $\theta'_c \approx \theta_c$, as $d(\Delta \theta)/dt \rightarrow \infty$. Estimates show that due to the difference in $d(\Delta \theta)/dt$, $\theta'_c$ as obtained in the current-phase measurement is reduced from the appropriate $\theta'_c$ in I-V measurement by an amount $\sim \frac{\theta_c}{2}$, in reasonable agreement with experimental observations.

The intersect with the current axis of a line tangent to the I-V at large currents, the excess current $i_{ex}$, is often used to describe weak-link behavior. In the shunted junction model having a piecewise linear CPR, $i_{ex}$ is obtained from $\theta'_c$ by

$$i_{ex} = i_c([\theta'_c - \pi]/\theta'_c).$$

In Fig. 1(a) $\theta'_c$ is 0.90$\times$2$\pi$ so that $i_{ex} = 0.44i_c$. In Fig. 1(b) for currents $\geq 150 \mu$A the I-V begins to turn upward, an effect probably due to heating. Because of this upward curvature we have fitted Eq. (1) to the data only for $i \leq 150 \mu$A. The best fit to the data was obtained with $\theta'_c = 1.45 \times 2\pi$ so that $i_{ex} = 0.66i_c$. The increase in the best fit of $\theta'_c$ from Fig. 1(a) to Fig. 1(b) agrees very well with the change in $\theta'_c$ with $i_c$ that was observed in the current-phase measurements.

In general, $i_{ex}$ was found to be either $\frac{1}{2}i_c$ or $\frac{3}{2}i_c$, depending on the weak-link CPR. Since it has been found that for UT microbridges the CPR depends strongly on phase winding in the electrodes, this result shows that one cannot determine weak-link response characteristics solely from a consideration of an isolated phase slip center of dimensions $\sim \xi$, the coherence length. Rather one must consider the dynamics of the order parameter in all the weak-link structure that contributes significantly to the CPR.

I-V's and CPR's were measured for proximity bridges formed by the overlay of 1000-Å-thick Sn strips on 250-Å Au strips. The widths of the strips were such that the resultant bridges were 1 $\mu$m long and 8 $\mu$m wide. I-V measurements on proximity bridges were made for temperatures $T$ with $T - T_c \leq 0.1$ K, where $T_c$ is the temperature for the onset of measurable supercurrent flow across the bridge. Large critical currents, resulting in hysteretic heating effects, prevented I-V studies at temperatures near $T_c - 2$ K, the superconducting transition temperature of the Sn-Au bilayer region. Thus, the I-V measurements were made in the temperature regime where supercurrent flow across the bridge is due to S-N-S tunneling through the bilayer region. In this regime the measured CPR's are invariably sinusoidal. Our I-V measurements show the applicability of a sinusoidal CPR at finite voltages, a result which is consistent with the recent report of Ganz and Mercereau on the nature of self-induced steps in proximity-bridge I-V curves.

Figure 2(a) shows the I-V of a proximity bridge measured one day after the bridge was fabricated. There is excellent agreement between the AH theory and the data. Because of the sinusoidal CPR, the AH theory predicts $i_{ex} = 0$, in agreement with the data. For $i \geq 200 \mu$A deviations from the AH theory were found with the measured I-V developing upward curvature. This curvature is probably due to heating or relaxation time effects.

After one month aging at room temperature we re-measured the I-V of the bridge in Fig. 2(a) and obtained the result shown in Fig. 2(b). At low currents and voltages a good fit of the AH theory was still found, but significant upward curvature began to develop for $i \geq 15 \mu$A where $V = 1 \mu$V. At these very low power levels heating effects should be minimal, indicating...
that the higher differential resistance at larger currents is most likely due to a relaxation time effect. Presumably during the aging process the metallurgical properties of the bridge changed, altering relevant relaxation times.

When driven to high enough current levels, all our proximity-bridge I-V's exhibited upward curvature. The onset current for the appearance of this curvature varied from sample to sample, but it was usually between 10 and 200 μA. Before the upward curvature is evident the I-V goes through an inflection point so that there is a region of the I-V which is nearly linear. If the onset of upward curvature is at currents only a few times Ic, such as in Fig. 2(b), a tangent drawn through the inflection point intersects the current axis at some positive value, giving the appearance of an excess current. We suggest that the reported excess currents in proximity bridges, which have in part led to a proposed finite-voltage CPR, 14 \( i = i_c \left[ \frac{1}{2} + \frac{1}{2} \cos(\Delta \theta) \right] \), may be due to the distortion of the I-V by this upward curvature.

We have found strong evidence for the validity of a weak-link model which includes the proper dc current-phase relation and a shunt resistance. We find excellent fits of proximity-bridge I-V's to the AH theory which includes fluctuation effects in a shunted junction model with a sinusoidal CPR. Our measurements on uniform-thickness Sn bridges show that the I-V's are well described by a shunted junction model with a piecewise linear CPR with varying critical phase \( \theta_c \).

In both types of bridges at higher current and voltage levels where heating and/or relaxation times become important we see departures from the shunted junction picture which appear as an increase in the differential resistance.

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Effect of stress on the critical current of Nb₃Sn multifilamentary composite wire*

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A critical-current study of flexible Nb₃Sn multifilamentary composite wires has been conducted at 4 K in magnetic fields to 90 kOe (7.2 x 10⁶ A/m) while the wire is subjected to high mechanical stresses. The results show that at stresses above 1-2 x 10⁶ Pa (strains of 0.1-0.2%) the critical current is significantly degraded, with the magnitude of the reduction dependent on reinforcement techniques used in the wire's construction. The effect increases with magnetic field and results in the introduction of significant resistance at current levels well below the zero-stress critical current. Design considerations for the use of Nb₃Sn wires in the high-stress environments of large-scale superconducting magnets are discussed.

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Large-scale superconducting magnets are being developed for such applications as superconducting rotating machinery, CTR plasma confinement, and energy storage. In these large magnet systems, magnetomechanical forces generated when the magnet is energized can subject the superconducting windings to high static loads. Recently, it was shown that the application of such loads to ductile NbTi:Cu multifilamentary wire at cryogenic temperatures results in a reversible but significant decrease in critical current, amounting to about 30% at high stresses. Here, initial data are presented which show the effects of stress in experi-

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