Superconducting weak-link current-phase relations


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Direct measurements have been made of the current-phase relation (CPR) of photolithographic tin microbridges, scribed indium microbridges, proximity bridges, and point contacts. Hysteretic current-phase relations have been observed in uniform-thickness tin microbridges, while proximity bridges have sinusoidal or nearly sinusoidal current-phase relations.

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We have made direct high-resolution measurements of the current-phase relation (CPR) of various types of common weak links. Included in the study were (i) photolithographic thin-film microbridges, (ii) scribed thin-film microbridges, (iii) proximity bridges, 1 and (iv) point contacts. Results include the first observation of distinctly nonsinusoidal CPR's as well as the measurement of a sinusoidal relation in proximity bridges where a different CPR has been suggested to hold. 1

The CPR is perhaps the most crucial determinant of superconducting weak-link behavior and yet, experimentally, it is usually the least well-known weak-link property. While the most common approach in analyzing weak-link experiments is to assume a sinusoidal CPR, 2-4 experimental and theoretical studies indicate that for many weak links the CPR should be distinctly nonsinusoidal. 1,5-7

Apart from its intrinsic interest, it is of considerable scientific and technical importance to measure directly weak-link CPR's. The recent effort to study the Josephson quasiparticle interference phenomenon is but one example where the analysis of a weak-link experiment is based on the assumption of a particular CPR. 8-10 It has also been shown recently that the performance of weak-link devices can be seriously degraded by nonsinusoidal CPR's. 11,12

The experimental technique consists of closing a low-inductance superconducting ring, \( L = 4 \times 10^{-10} \) H, with the weak link of interest. An external magnetic flux \( \psi_0 \) is applied to the ring. The flux \( \phi \) within the ring is monitored with a superconducting flux detector. Fluid flux quantization requires that the phase difference \( \Delta \phi \) across the weak link satisfy \( \Delta \phi = -2 \pi \phi / \phi_0 \) (modulo \( 2\pi \)). Here \( \phi_0 \) is the flux quantum, \( \phi_0 = 2.1 \times 10^{-7} \text{Gcm} \). The current flowing in the ring is \( i = (\phi - \phi_c) / L \). Thus by monitoring \( \phi \) as a function of \( \phi_c \), information concerning the CPR \( i(\Delta \phi) \) can be obtained: \( i = i(\Delta \phi) = i_c(\Delta \phi) \), where \( i_c \) is the weak-link supercurrent, \( i_c \) is the weak-link critical current, and \( f(\Delta \phi) \) is an odd function with a maximum amplitude of unity, whose value is unchanged by a shift of \( 2\pi \) in \( \Delta \phi \).

The flux detector is calibrated and nulled against direct pickup of the applied flux by cooling the superconducting ring well below \( T_c \), where \( L i_c \gg \phi_0 \). At this point \( \phi \) is essentially independent of \( \phi_c \) for \( \phi_c \sim \phi_0 \) which permits an accurate nulling of the detector. The periodic changes in \( \phi \) that are observed for larger \( \phi_c \) are then assumed to be exactly one flux quantum which accurately calibrates the detector.

The obtainable current-phase information depends on the value of \( i_c \) and on the form of \( f(\Delta \phi) \). If \( i_c \ll \phi_0 / L \) and if \( f(\Delta \phi) \) is single valued, the \( \phi-\phi_c \) curve is also single valued, permitting complete determination of the CPR. 13 Once corrections have been made for fluctuation effects. 14

If \( i_c > \phi_0 / L \) or if \( f(\Delta \phi) \) is multivalued, then the \( \phi-\phi_c \) curves are hysteretic. An increasing \( \phi_c \) results in an increasing \( i_c \) flowing in the weak link until a critical phase angle \( \phi_c \) is approached, where \( f(\phi_c) = 1 \) and \( i_c = i_c \). Near this point an irreversible transition is made to an adjacent fluxoid state, thermal fluctuations causing the transition to occur somewhat before \( i_c \) is reached. Provided \( i_c \) is fairly large (\( \sim 20 \mu A \)), 15 most of the positive slope segments of the CPR can be mapped, and a lower bound for \( \phi_c \) can be established which is close to the true value.

It should be mentioned that, as a result of the calibration and nulling procedure described above, in both measurement regimes any temperature-independent contribution to the CPR, such as a temperature-independent kinetic inductance term, will not be included in the measured CPR. If, for some unexpected reason, such terms are not negligible, the correct CPR for the weak link would have a lesser initial slope and a greater critical phase angle \( \phi_c \) than is experimentally observed.

Typical results for the CPR of uniform-thickness thin-film tin microbridges are shown in Fig. 1. For these specimens, the thickness of the microbridges and adjacent electrodes was 1000 Å, bridge widths were \( \sim 1 \mu \), bridge lengths varied from 1 to 250 \( \mu \), and edge definition was \( \sim 1000 \) Å. All of the microbridges tested were found to have nonsinusoidal CPR's qualitatively similar to those shown in Fig. 1 with the measurements being made from just below \( T_c \) of the microbridge, where \( i_c < 0.5 \mu A \), to \( T_{\phi} \), where \( i_c > 100 \mu A \). 16 While \( \phi_c \) decreased for \( T < T_c \), even for \( T_{\phi} \), the CPR less than \( 10^{-2} \), where the coherence length is greater than the shorter bridges, \( f(\Delta \phi) \) remained very nonsinusoidal. Only when bridges were longer than \( \sim 25 \mu \) did \( f(\Delta \phi) \) vary significantly with \( L \). For example, with \( i_c < 10 \mu A \)
The critical phase angle \( \theta_c \) was \( \sim \frac{\pi}{2} \) for the shorter microbridges, while increasing to more than \( 4\pi \) for the 250-\( \mu \)-long bridge.

We attribute the nonsinusoidal behavior of these microbridges to a phase gradient or "phase-winding" in the electrodes leading to the bridge. A simple calculation can be used to estimate this effect. Consider two semicircular electrodes of radius \( R \) connected by a microbridge of length \( L \) and width \( W \) with both bridge and electrodes having film thickness \( t \). The product of critical current density \( j_c \) and \( t \) is constant throughout the thin-film structure. The ratio of electrode phase-winding to bridge phase-winding, \( \Delta \phi_c / \Delta \phi_B \), is

\[
\frac{\Delta \phi_c}{\Delta \phi_B} = \frac{(2/\pi) \ln(2R/W)}{(L/W)}.
\]

If \( R = 1 \) mm and \( L = W = 1 \) \( \mu \), then \( \Delta \phi_c / \Delta \phi_B < 5 \). Thus the total phase gradient across the entire weak-link structure is dominated by electrode phase-winding until \( L \gg W \). These results indicate that except for very small narrow microbridges the electrode contribution to the complete CPR of uniform-thickness microbridges is very important which tends to make \( f(\Delta \theta) \) for such weak-link structures distinctly nonsinusoidal.

Figure 2 shows a measured CPR for an In thin-film microbridge, \( \sim 0.7 \mu \) long and \( \sim 0.5 \mu \) wide, prepared by the double-scribing technique of Gregers-Hansen and Levinsen. All of the scribed microbridges tested had either sinusoidal or nearly sinusoidal CPR's for \( T \lesssim 0.98T_c \). For \( T > 0.98T_c \), \( \theta_c \) became as large as \( 1.5\pi \) for the bridge of Fig. 2. For other smaller bridges \( \theta_c < \pi \) for all \( T \). This marked difference in behavior compared to our tin bridges can be attributed to the considerably reduced thickness of the scribed bridges relative to the electrodes and the possibility that the scribed bridges are small enough to have a reduced mean free path in the bridge region, reducing \( j_c \). Thus \( j_c \) is significantly less in the bridge than in the electrodes, greatly reducing the effect of electrode phase-winding.

Of the tin-gold proximity bridges that were tested, some had perfectly sinusoidal CPR's; none had a hysteretic \( f(\Delta \theta) \). Although this result is in apparent agreement with a recent measurement of Ganz and Mercereau, it is not in agreement with the empirical CPR, \( f(\Delta \theta) = \frac{1}{2}(1 + \cos \Delta \theta) \), that has been proposed to explain the behavior of these weak links at finite voltages.

The proximity bridges were formed by overlaying a 250-\( \AA \)-thick gold strip with a 1000-\( \AA \) tin strip. Bridges were fabricated with lengths and widths between 1 and 8 \( \mu \). The only apparent effect of increased bridge width was an increase in critical current at fixed \( T \) with no effect on \( T_c \), the onset temperature for supercurrent flow. The primary length effect was a modification of the bridge \( T_c, T_c \) ranged from 3.2 K in a 1-\( \mu \)-long bridge down to \( \approx 2 \) K in an 8-\( \mu \)-long bridge. CPR's were only slightly affected by length variations. Near \( T_c \), all the bridges had sinusoidal CPR's, but at temperatures \( \approx 2 \) K bridges 3 \( \mu \) and longer showed \( \theta_c \) increasing

<table>
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<th>Weak link</th>
<th>Range of ( \theta_c )</th>
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<td>Tin microbridges</td>
<td>( &gt; 0.7\pi \rightarrow 4\pi )</td>
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<tr>
<td>Scribed In bridges</td>
<td>( \pi/2 - 3\pi/2 )</td>
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Microfabrication of circuits for magnetic bubbles of diameter 1 \( \mu \text{m} \) and 2 \( \mu \text{m} \)


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We describe microfabrication techniques for Permalloy circuits for magnetic bubbles of diameter 1 \( \mu \text{m} \) and 2 \( \mu \text{m} \). Patterns are defined by electron beam on an x-ray mask. X-ray lithography and ion milling are employed to replicate the pattern in Permalloy.

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Magnetic bubble memory circuits of high bit density require circuit patterns with minimum features of sub-micron size. For example, a circuit for bubbles of diameter 1 \( \mu \text{m} \) requires gaps between neighboring Permalloy elements in the range 0.25–0.5 \( \mu \text{m} \), depending upon the circuit, and represents an information density of more than 10^8 bits/cm^2. Because of the small features of the circuit the patterns cannot be defined by standard optical lithography. Such resolution is attainable with electron beam lithography. However, because of the long times required to write such patterns over large areas, it is not practical to write on each slice individually.

X-ray lithography is a technique complementary to electron beam lithography in which a high-resolution mask is generated by electron beam writing and is used to pattern slices by x-ray exposure through the mask.\(^1\)\(^2\) The technique is especially suited to magnetic bubble memory circuits, since the high-density pattern is contained in a single level and no registration of multiple levels to submicron tolerances is necessary. In this letter, we report preliminary results of our evaluation of this lithographic technique together with ion milling for the construction of operational circuits for bubbles with diameter as small as 1 \( \mu \text{m} \). Device characteristics will be described elsewhere.\(^3\)

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\(^1\)L. P. Khanam, Phys. Rev. B (to be published).


\(^1\)A load-line analysis shows that the \( \phi - \Delta \phi \) magnetization curve is single valued if \( L_d < (\phi/2\pi)/(-\text{min}(\phi, \Delta \phi)/\phi(\Delta \phi)) \).


\(^1\)Measurements at \( I = 100 \mu \text{A} \) were usually not possible because of erratic multiple fluxoid transitions.

